Algebraic Geometry I Prof. Dr. Ulrich Görtz
WS 2018/19

Problem sheet 13

Due date: Jan. 22, 2018.

Problem 53

Let Y be a scheme, and let $i\colon Z \subseteq Y$ be a subscheme. Prove that the associated sheaf homomorphism $\mathscr{O}_Y \to i_*\mathscr{O}_Z$ is surjective.

Problem 54

Let Y be a scheme, and let $i\colon Z \subseteq Y$ be a subscheme. Prove that a morphism $f\colon X \to Y$ of schemes factors through the subscheme Z if and only if the following conditions are satisfied:

(i) $f(X) \subseteq Z$ (set-theoretically),

(ii) $f^\flat\colon \mathscr{O}_Y \to f_*\mathscr{O}_X$ factors through the surjective homomorphism $\mathscr{O}_Y \to i_*\mathscr{O}_Z$.

Prove that (i) implies (ii) if Z is an open subscheme, or if X is reduced.

Problem 55

Let $n \geqslant 1$ be an integer and set $X = \operatorname{Spec} \mathbb{Q}[S,T]/(S^n + T^n - 1)$. Translate the condition that there exist nonzero integers $x, y, z \in \mathbb{Z}$ with $x^n + y^n = z^n$ into a statement about $X(\mathbb{Q})$.

Problem 56

Let k be a field, k^{sep} a separable closure, $\Gamma := \operatorname{Gal}(k^{\mathrm{sep}}/k)$ the group of all k-automorphisms of k^{sep}, and let X be a k-scheme locally of finite type. Show that for all $x \in X(k^{\mathrm{sep}})$ the Γ-orbit of x in $X(k^{\mathrm{sep}})$ is finite.